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# Method for aerodynamic unsteady forces time calculations on an F/A-18 aircraft

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## ABSTRACT

In this paper, a new original method based on the least squares method is presented for the conversion of unsteady aerodynamic forces from frequency into Laplace domain, in which the error is written in an analytical form as a function of the Laplace variable, similar to the analytical form of the aerodynamic forces calculated by use of the least squares method. This method is applied on an F/A-18 aircraft (14 symmetric and 14 anti-symmetric modes) for one Mach number and for a set of 14 reduced frequencies. Two different types of results are obtained and analysed: aerodynamic force approximations in the Laplace domain and flutter speeds and frequencies values. For a better comparison of these results, different lag term numbers are used. Results obtained by this new method are better in terms of execution speed and precision than the results obtained by use of the least squares method.

## NOMENCLATURE

| | |
|---|---|
| $A_e\,(k)$ | aerodynamic influence coefficient matrix |
| $b$ | wing span |
| **C** | generalised damping matrix |
| DLM | doublet lattice method |
| $e_1$ | error between the aerodynamic forces calculated with the LS method *versus* the aerodynamic forces in calculated with the DLM method |
| $e(\bar{s})$ | error between the aerodynamic forces calculated with the LS method *versus* the aerodynamic forces in calculated with the CLS method |
| *Err* | error reduction rate |
| ISAC | interaction of structures, aerodynamics and controls |
| $J$ | quadratic optimisation criteria |
| **K** | stiffness generalised matrix |
| $k_i$ | reduced frequencies |
| LS | least squares method |
| **M** | mass generalised matrix |
| $P(t)$ | external forcing function |
| $Q_{CLS}(\bar{s})$ | aerodynamic forces approximations with CLS method |
| $Q_{LS}(\bar{s})$ | aerodynamic forces approximations with LS method |
| **q** | displacement vector |
| $q_{dyn}$ | dynamic pressure |
| $Q(k)$ | forces aérodynamiques en fonction de la fréquence réduite |
| $Q(s)$ | aerodynamic forces in the Laplace domain |
| $\bar{s}$ | normalised Laplace variable |
| $S$ | wing surface |
| STARS | structural analysis routines |
| $V$ | true airspeed |

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## 1.0 INTRODUCTION

The aeroservoelasticity concerns the interactions between following disciplines: aerodynamics, aeroelasticity and servo-controls mainly on a fly-by-wire aircraft equipped with active control systems. The aerodynamic unsteady forces are determined in the frequency domain with the doublet lattice method (DLM) in the subsonic regime or with the constant pressure method (CPM) in the supersonic regime by use of a finite element software such as Nastran or STARS. One main aspect of the aeroservoelasticity concerns the conversion of these unsteady generalised aerodynamic forces from the frequency domain into the Laplace domain by use of various methods. The three classical methods mainly used for this type of conversion are: least square (LS), matrix Padé (MP) and minimum state (MS)[1-5]. In this paper, a new original method is developed which is based on the least square method and gives better results in terms of accuracy and execution time. We present in this section the bibliographical research on the classical methods.

The LS and MS methods were further improved and renamed extended LS[6] methods (ELS)[6] and extended modified matrix Padé[7] method (EMMP). Different conditions (restrictions) were imposed on these approximations so that they pass through certain points. These constraints were imposed at zero and at two other chosen points. For example, the first point could represent the estimated flutter frequency and the second point could represent the gust frequency.

Poirion[8] used several MS approximations, obtained for several fixed Mach numbers, and a spline interpolation method for Mach number dependence. Approximations with his method are further determined for any couple ($k$, M), where $k$ is the reduced frequency and M is the Mach number.

Botez and Cotoi[9] showed a new approach based on a precise Padé approximation. Four order reduction methods for the last term of the approximation were used, and this last term was seen as a transfer function of a linear system. The approximation error for this new method was found to be 12-40 times less than for the MS method for the same number of augmented states and was dependent on the choice of the order reduction method. However, this method is very expensive in terms of computing time compared to the MS method.

Cotoi, Dinu and Botez[10] presented another method to approximate the generalized aerodynamic forces by use of Chebyshev polynomials and their orthogonality properties. A comparison of this new method with the Padé method used to calculate an approximation of the generalized aerodynamic forces was presented. This new approximation method gave excellent results with respect to the Padé method.

Hiliuta, Botez and Brenner[11] applied a combination of ‘pchip’ and ‘fuzzy clustering techniques’ for the interpolation of unsteady forces calculated for a range of non-evenly spaced reduced frequencies. However, if the range of reduced frequencies was evenly spaced, the results were obtained by use of the least square method. With this new method (efficient mainly for a range of non-evenly reduced frequencies), the approximations of these unsteady generalised forces remained in the frequency domain, and in order to obtain their approximations in the Laplace domain, it was necessary to apply a classical method such as least squares or minimum state.

In this paper, a new method based on the least square method is presented. The new feature presented in this paper is that the error between the results given by the new method and that given by the LS method is written in an analytical form similar to that of the aerodynamic forces calculated by the LS method. This method is applied on the aerodynamic force approximations for one Mach number and 14 reduced frequencies. The results, expressed in the form of aerodynamic force approximations, and the flutter speeds and frequencies obtained by this new method are much better than the results obtained with the least squares method.

## 2.0 EQUATIONS OF MOTION

The equations of motion for a flexible aircraft structure are expressed by the following matrix equation[12]:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + q_{dyn}\mathbf{A}_e(k)\mathbf{q} = \mathbf{P}(t) \qquad \ldots (1)$$

In Equation (1), the matrices **M**, **C** and **K** are the generalised mass, damping and stiffness matrices, $Q_{dyn} = 0{\cdot}5\ \rho V^2$, is the dynamic pressure, where $\rho$ is the air density and $V$ is the true airspeed, $k = \omega b/V$ is the reduced frequency where $\omega$ is the natural frequency, $b$ is the wing semi-chord length, $A_e\ (k)$ is the aerodynamic influence coefficient matrix for a given Mach number M and a set of reduced frequencies, $\mathbf{q}$ is the displacement vector and $\mathbf{P}(t)$ is the external forcing function. By pre-multiplying both sides of Equation (1) by $\Phi^T$ and $\Phi$, the generalised equation of motion (1) becomes:

$$\hat{\mathbf{M}}\ddot{\eta} + \hat{\mathbf{C}}\dot{\eta} + \hat{\mathbf{K}}\eta + q_{dyn}\mathbf{Q}(k)\eta = \hat{\mathbf{P}}(t) \qquad \ldots (2)$$

where $\hat{\mathbf{M}} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}$, $\hat{\mathbf{C}} = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}$, and so forth.

The unsteady aerodynamic generalised forces **Q** are calculated using STARS code[12] for one Mach number and a range of 14 reduced frequencies, $k$’s, on the F/A-18 aircraft. The equations of motion in the frequency domain (2) are converted to the Laplace domain, and the following equation is obtained:

$$\left[\hat{\mathbf{M}}s^2 + \hat{\mathbf{C}}s + \hat{\mathbf{K}} + q_{dyn}\mathbf{Q}_{LS}(\bar{s})\right]\eta(s) = \hat{\mathbf{P}}(s) \qquad \ldots (3)$$

where we denote $\bar{s} = sb/V$ as the normalised Laplace variable. The generalised aerodynamic forces $\mathbf{Q}(k)$ given in Equation (2) are first approximated in the Laplace domain by use of the least squares LS method (subscript $LS$), and are also denoted by $Q_{LS}(\bar{s})$ as in Equation (3).

## 3.0 DESCRIPTION OF OUR METHOD

The approximation of unsteady aerodynamic forces in the Laplace domain by the LS method $\mathbf{Q}_{LS}(\bar{s})$ is expressed in the following form:

$$Q_{LS}(\bar{s}) = A_0^{LS} + A_1^{LS}\bar{s} + A_2^{LS}\bar{s}^2 + \frac{A_3^{LS}}{\bar{s}+b_1^{LS}}\bar{s} + \frac{A_4^{LS}}{\bar{s}+b_2^{LS}}\bar{s} + \ldots + \frac{A_{n+2}^{LS}}{\bar{s}+b_n^{LS}}\bar{s} \qquad \ldots (4)$$

where $A_i^{LS}$ are the $n$ + 3 estimated matrices and $B_n^{LS}$ are the $n$ estimated lag terms.

In the least square LS method, we need to increase the number of lag terms in order to find the best values of the aerodynamic unsteady force approximations in the Laplace domain $Q_{LS}(\bar{s})$ close to their initial values in the frequency domain $Q(k)$. In this paper, we present a new method, called the CLS method, which minimises the error between the aerodynamic forces $\mathbf{Q}_{CLS}(\bar{s})$ calculated and their approximation $\mathbf{Q}_{LS}(\bar{s})$ in the Laplace domain. This error is also expressed in the same analytical form as the aerodynamic forces determined by the LS method. The main advantage of the new method presented here is that it can be obtained with fewer lag terms and, thus, the execution time can be reduced.

The matrix error $e_1$ is expressed as the difference between the unsteady aerodynamic forces calculated in the frequency domain $Q(k)$ and their approximations by the least squares LS method in the normalised Laplace domain $\bar{s}$, $\mathbf{Q}_{LS}(\bar{s})$:

$$e_1 = \mathbf{Q}(k) - \mathbf{Q}_{LS}(\bar{s}) \qquad \ldots (5)$$

The difference between the aerodynamic forces $\mathbf{Q}_{CLS}(\bar{s})$ approximated with the new method and the approximations of aerodynamic forces calculated by the classical LS method $\mathbf{Q}_{LS}(\bar{s})$ may be written as an error dependent on $\bar{s}$, denoted as $e(\bar{s})$:

$$\mathrm{e}(\bar{s}) = \mathbf{Q}_{CLS}(\bar{s}) - \mathbf{Q}_{LS}(\bar{s}) \qquad \text{. . . (6)}$$

This error function $e(\bar{s})$ is next approximated using the same analytical form as that given by the least squares LS method. This form is similar to the analytical form of $\mathbf{Q}_{LS}(\bar{s})$ given by Equation (4):

$$e(\bar{s}) = \mathrm{A}_0^{err} + \mathrm{A}_1^{err}\bar{s} + \mathrm{A}_2^{err}\bar{s}^2 + \frac{\mathrm{A}_3^{err}}{\bar{s}+b_1^{err}}\bar{s} + \frac{\mathrm{A}_4^{err}}{\bar{s}+b_2^{err}}\bar{s} + \dots \frac{\mathrm{A}_{n+2}^{err}}{\bar{s}+b_n^{err}}\bar{s} \qquad \text{. . . (7)}$$

where the superscript *err* denotes the LS analytical error form.

A first assumption to be considered to simplify this new method is to configure the same number of lag terms *n* in both the LS and CLS methods, which can be written as follows:

$$b_1^{err} = b_1^{LS},\ b_2^{err} = b_2^{LS},\ \dots,\ b_n^{err} = b_n^{LS} \qquad \text{. . . (8)}$$

The lag terms $b_{1,2,3..}^{err}$ and $b_{1,2,3..}^{LS}$ will be denoted by $b_{1,2,3...}$ as the superscripts *err* and *LS* have been dropped out from Equations (7) and (4).

For each element $e^{r,c}$ of the error matrix (**e**), the minimisation criteria $J^{r,c}$ is defined by use of the following equation:

$$J^{r,c} = \sum_1^l \left[ e^{r,c} - \left( A_0^{err}(r,c) + A_1^{err}(r,c)\bar{s} + A_2^{err}(r,c)\bar{s}^2 + \sum_{i=1}^{n} \frac{A_{i+2}^{err}(r,c)}{\bar{s}+b_i}\bar{s} \right)\right]^2 \qquad \text{. . . (9)}$$

where *r* and *c* indicate the row and column indices, and *l* is the number of reduced frequencies, which is equal to 14 in this paper. The (*r*,*c*) superscript in the following equations (demonstrating our new method) will later be dropped out for notation simplicity. The partial derivatives of $J^{r,c} = J$ with respect to each unknown factor $A_{1,2..}^{err}(r,c) = A_{1,2,3}^{err}$ will be set to zero in order to minimise the optimisation criterion $J^{r,c}$, which may be written in the following form:

$$\frac{\partial J}{\partial A_{0,1,2,...}^{err}} = 0 \qquad \text{. . . (10)}$$

$$\frac{\partial J}{\partial A_0^{err}} = (-2)\sum_1^l \left[ e - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0 \qquad \text{. . . (11)}$$

$$\frac{\partial J}{\partial A_1^{err}} = (-2)\sum_1^l \bar{s}\left[ e - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0$$

$$\frac{\partial J}{\partial A_2^{err}} = (-2)\sum_1^l \bar{s}^2\left[ \mathrm{e} - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0$$

$$\frac{\partial J}{\partial A_3^{err}} = (-2)\sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_1}\right)\left[ e - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0$$

$$\frac{\partial J}{\partial A_4^{err}} = (-2)\sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_2}\right)\left[ e - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0$$

$$\frac{\partial J}{\partial A_{n+2}^{err}} = (-2)\sum_1^l \left(\frac{s}{s+b_n}\right)\left[ e - \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)\right] = 0$$

which may also be written as follows:

$$\sum_1^l e = \sum_1^l \left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)$$

$$\sum_1^l \bar{s}e = \sum_1^l \bar{s}\left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)$$

$$\sum_1^l \bar{s}^2 e = \sum_1^l \bar{s}^2\left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right) \qquad \text{. . . (12)}$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_1} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_1}\right)\left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_2} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_2}\right)\left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_n} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_n}\right)\left( A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^{n}\frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s}\right)$$

which then can be written in the following form:

$$\sum_1^l e = A_0^{err}\sum_1^l 1 + A_1^{err}\sum_1^l \bar{s} + A_2^{err}\sum_1^l \bar{s}^2 + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}}{\bar{s}+b_i}\right)\right]$$

$$\sum_1^l \bar{s}e = A_0^{err}\sum_1^l \bar{s} + A_1^{err}\sum_1^l \bar{s}^2 + A_2^{err}\sum_1^l \bar{s}^3 + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n}\right)\right]$$

$$\sum_1^l \bar{s}^2 e = A_0^{err}\sum_1^l \bar{s}^2 + A_1^{err}\sum_1^l \bar{s}^3 + A_2^{err}\sum_1^l \bar{s}^4 + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n}\right)\right]$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_1} e = A_0^{err}\sum_1^l \frac{\bar{s}}{\bar{s}+b_1} + A_1^{err}\sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} + A_2^{err}\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} \cdot + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_1)}\right)\right] \qquad \text{. . . (13)}$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_2} e = A_0^{err}\sum_1^l \frac{\bar{s}}{\bar{s}+b_2} + A_1^{err}\sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} + A_2^{err}\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} \cdot + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_2)}\right)\right]$$

$$\sum_1^l \frac{\bar{s}}{\bar{s}+b_n} e = A_0^{err}\sum_1^l \frac{\bar{s}}{\bar{s}+b_n} + A_1^{err}\sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} + A_2^{err}\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} + \cdot + A_2^{err}\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} + \sum_1^n\left[ A_{n+2}^{err}\left(\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_n)}\right)\right]$$

The system of Equations (13) may be written as the following matrix product:

$$b = A\,x \qquad \text{. . . (14)}$$

where

$$b = \left[ \sum_1^l e \sum_1^l \bar{s}e \quad \sum_1^l \bar{s}^2 e \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} e \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} e \dots \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} e \right]^T \qquad \text{. . . (15)}$$

and

$a =$

$$\begin{bmatrix}
\sum_1^l 1 & \sum_1^l \bar{s} & \sum_1^l \bar{s}^2 & \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} \\
\sum_1^l \bar{s} & \sum_1^l \bar{s}^2 & \sum_1^l \bar{s}^3 & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} \\
\sum_1^l \bar{s}^2 & \sum_1^l \bar{s}^3 & \sum_1^l \bar{s}^4 & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} \\
\sum_1^l \frac{\bar{s}}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_1)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_1)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_1)} \\
\sum_1^l \frac{\bar{s}}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_2)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_2)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_2)} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\sum_1^l \frac{\bar{s}}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_n)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_n)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_n)}
\end{bmatrix} \quad \ldots (16)$$

The solution of the system of equations $x$ is obtained from Equation (14) and then expressed as:

$$x = A^{-1}b = \begin{bmatrix} A_0^{err} & A_1^{err} & A_2^{err} & A_3^{err} & A_4^{err} & \ldots & A_{n+2}^{err} \end{bmatrix}^T \quad \ldots (17)$$

Next, we use Equation (6) to obtain the approximation of unsteady aerodynamic forces by our new method, denoted here by $\mathbf{Q}_{CLS}(\bar{s})$:

$$Q_{CLS}(\bar{s}) = Q_{LS}(\bar{s}) + e(\bar{s}) \quad \ldots (18)$$

We replace $\mathbf{Q}_{LS}(\bar{s})$ and $\mathbf{e}(\bar{s})$ given by Equations (4) and (7) into Equation (18) and then $\mathbf{Q}_{CLS}(\bar{s})$ is written in the following form:

$$Q_{CLS}(\bar{s}) = (A_0^{LS} + A_0^{err}) + (A_1^{LS} + A_1^{err})\bar{s} + (A_2^{LS} + A_2^{err})\bar{s}^2 + \sum_{i=1}^{n} \frac{\bar{s}}{\bar{s} + b_i^{LS}} (A_{i+2}^{LS} + A_{i+2}^{err}) \quad \ldots (19)$$

Equation (19) can be expressed in the following shorter form given by the following Equation (20):

$$Q_{CLS}(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + \sum_{i=1}^{n} \frac{A_{i+2}}{\bar{s} + b_i^{LS}}\bar{s} \quad \ldots (20)$$

where $A_0 = A_0^{LS} + A_0^{err}$, $A_1 = A_1^{LS} + A_1^{err}$, $A_2 = A_2^{LS} + A_2^{err}$, …, $A_{n+2} = A_{n+2}^{LS} + A_{n+2}^{err}$.

## 4.0 RESULTS

The aerodynamic generalised forces were approximated using two methods, LS and our new method denoted as CLS, on an F/A-18 aircraft (symmetric and anti-symmetric modes) for the following flight condition: Mach number M = 1·6 and a number of 14 reduced frequencies $k$ = 0·01, 0·1, 0·2, 0·3, 0·4, 0·5, 0·59, 0·63, 0·67, 0·71, 0·77, 0·83, 0·91 and 1.

In order to compare the results obtained by the new method with the results from the LS method, we define the error reduction rate by the following equation:

$$Err = 100 * \text{abs}\left(\frac{Q_{LS} - Q_{CLS}}{Q_{LS}}\right) \quad \ldots (21)$$

which is calculated for the real and imaginary parts of aerodynamic forces *Q*_real and *Q*_imag and for the total aerodynamic forces *Q*_total, and where abs is the absolute value (see Figs 1 and 2 and Tables 1 and 2).

**Table 1**
**Error reduction rate for the F/A-18 anti-symmetric modes aerodynamic forces given under numerical form**

| Number of lag terms | Error reduction rate for *Q*_real (%) | Error reduction rate for *Q*_imag (%) | Error reduction rate for *Q*_total (%) |
|---|---|---|---|
| 1 | 72·65 | 74·83 | 73·93 |
| 2 | 82·80 | 76·28 | 80·07 |
| 3 | 83·01 | 80·41 | 81·69 |
| 4 | 89·91 | 88·33 | 89·24 |
| 5 | 89·25 | 88·46 | 88·88 |
| 6 | 95·99 | 92·76 | 94·63 |
| 7 | 87·33 | 84·39 | 85·49 |
| 8 | 53·18 | 54·50 | 54·26 |
| 9 | 25·46 | 43·78 | 36·63 |

**Table 2**
**Error reduction rate for the F/A-18 symmetric modes aerodynamic forces given under numerical form**

| Number of lag terms | Error reduction rate for *Q*_real (%) | Error reduction rate for *Q*_imag (%) | Error reduction rate for *Q*_total (%) |
|---|---|---|---|
| 1 | 66·30 | 67·98 | 67·58 |
| 2 | 78·65 | 74·57 | 77·04 |
| 3 | 75·00 | 69·37 | 73·08 |
| 4 | 76·90 | 81·25 | 79·33 |
| 5 | 86·26 | 82·86 | 85·20 |
| 6 | 67·36 | 69·15 | 68·96 |
| 7 | 71·75 | 78·06 | 75·34 |
| 8 | 63·04 | 74·94 | 70·03 |
| 9 | 55·72 | 61·11 | 59·28 |

For the anti-symmetric modes of an F/A-18 aircraft, Table 1 shows, from the first to the fourth columns, the number of lag terms, the error reduction for the aerodynamic forces real part, the error reduction for the aerodynamic forces imaginary part and the error reduction for the total aerodynamic forces. The results presented numerically in Table 1 are represented in bar form in Fig. 1.

For the symmetric modes of an F/A-18 aircraft, Table 2 shows, in columns 1 to 4, the number of lag terms, the error reduction for the aerodynamic forces real part, the error reduction for the aerodynamic forces imaginary part, and the error reduction for the total aerodynamic forces, respectively. The results presented numerically in Table 2 are displayed in bar form in Fig. 2.

As seen in Table 1 and its associated Fig. 1, we found that the aerodynamic forces error reduction rate (real, imaginary and total) is much smaller with our new CLS method compared to the least square LS method. The reduction rate for the error ranges from 25·46% to 95·99%, with the highest reduction rate at six lag terms in the case of the F/A-18 anti-symmetric modes. For the F/A-18 symmetric modes, the highest error reduction rate occurs at five lag terms, as shown in Table 2 and its associated Fig. 2.

The second portion of our results shows the flutter equivalent air speeds denoted by EAS and frequencies denoted by $f$ calculated by the LS method and by our method with respect to the flutter speeds and frequencies calculated by use of the *pk* standard method for lag terms number from 1 to 9. Equations (22.1) and (22.2) express these differences under the following percentage form, similar to the percentage form of the previous Equation (21):

$$EAS_{LS} = 100 * \text{abs}\left(\frac{EAS_{LS} - EAS_{pk}}{EAS_{pk}}\right) \text{ and}$$

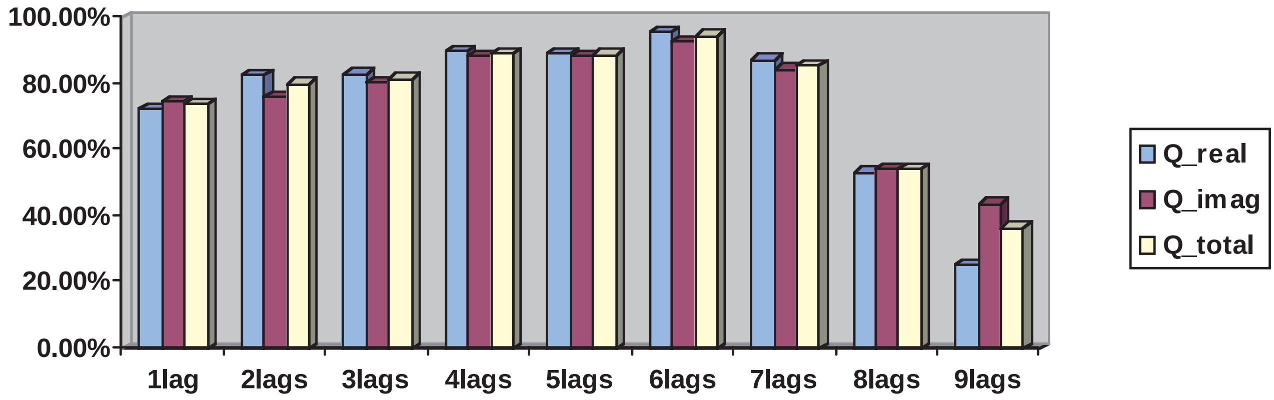


Figure 1. Error reduction rate as a percentage (%) for the F/A-18 anti-symmetric modes aerodynamic forces given under visual form.

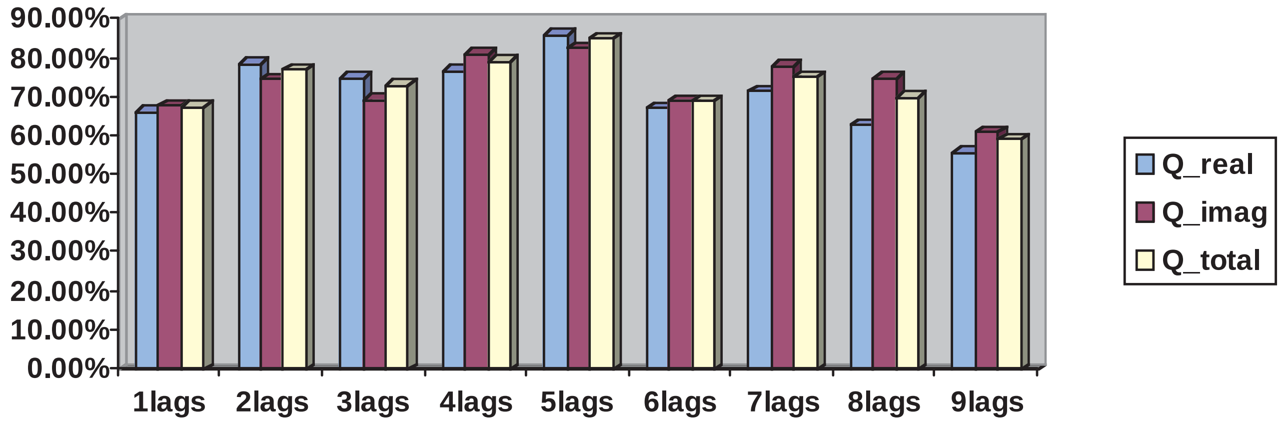


Figure 2. Error reduction rate as a percentage (%) for the F/A-18 symmetric modes aerodynamic forces given under visual form.

**Table 3**
**Relative error of the first flutter speeds and frequencies calculated by the LS and CLS methods for a set of 1 to 9 lag terms with respect to speeds and frequencies calculated by the *pk* standard method, Case: F/A-18 symmetric modes and M = 1·6**

| Number of lag terms | LS method $EAS_{LS}$ (%) | $f_{LS}$ (%) | CLS method $EAS_{CLS}$ (%) | $f_{LCS}$ (%) |
|---|---|---|---|---|
| 1 | 0·28 | 0 | –0·002 | 0 |
| 2 | 0·07 | 0 | –0·02 | 0 |
| 3 | 0·39 | 0·14 | –0·02 | 0 |
| 4 | 0·39 | 0·14 | –0·01 | 0 |
| 5 | 0·05 | 0 | –0·01 | 0 |
| 6 | –0·02 | 0 | –0·02 | 0 |
| 7 | 0·05 | 0 | –0·01 | 0 |
| 8 | –0·02 | 0 | –0·01 | 0 |
| 9 | –0·02 | 0 | –0·004 | 0 |

$$EAS_{CLS} = 100 * \text{abs}\left(\frac{EAS_{CLS} - EAS_{pk}}{EAS_{pk}}\right) \quad \ldots (22.1)$$

$$f_{LS} = 100 * \text{abs}\left(\frac{f_{LS} - f_{pk}}{f_{pk}}\right) \text{ and } \quad \ldots (22.2)$$

**Table 4**
**Relative error of the first flutter speeds and frequencies calculated by the LS and CLS methods for a set of 1 to 9 lag terms with respect to speeds and frequencies calculated by the *pk* standard method, Case: F/A-18 symmetric modes and M = 1·6**

| Number of lag terms | LS method EAS (%) | Frequency (%) | CLS method EAS (%) | Frequency (%) |
|---|---|---|---|---|
| 1 | –2·83 | –0·82 | –0·61 | 0 |
| 2 | –0·63 | –0·10 | –0·12 | 0·03 |
| 3 | –3·29 | –0·69 | 0·06 | 0·03 |
| 4 | –3·29 | –0·69 | 0·04 | 0·03 |
| 5 | –0·62 | –0·10 | 0·05 | 0·03 |
| 6 | –0·07 | 0·03 | 0·05 | 0·03 |
| 7 | –0·16 | 0·03 | 0·05 | 0·03 |
| 8 | –0·10 | 0·03 | 0·06 | 0·03 |
| 9 | –0·09 | 0·03 | 0·03 | 0·03 |

$$f_{CLS} = 100 * \text{abs}\left(\frac{f_{CLS} - f_{pk}}{f_{pk}}\right)$$

Tables 3 and 4 show the relative error of the first flutter speeds and frequencies calculated by the LS and CLS methods for a set of lag terms, with respect to speeds and frequencies calculated by the *pk* standard method, with M = 1·6; for symmetric modes (Table 3) and anti-symmetric modes (Table 4).

In Table 3, for the F/A-18 symmetric modes, the smallest equivalent airspeed percentage EAS (%) calculated by the LS method appears for six lag terms (–0·02). The EAS percentage closest) to this value, calculated by the CLS method, was found with two lag terms.

For the F/A-18 anti-symmetric modes (results displayed in Table 4), we found the smallest EAS (%) calculated by the LS method for six lag terms (–0·07 %). The closest EAS (%) to this value, calculated by the CLS method, was found for three lag terms (0·06 %).

## 5.0 CONCLUSIONS

The following conclusions were drawn:

- Generally, to obtain the flutter speed closest to the flutter speed calculated by the *pk* standard method, the new method requires fewer lag terms than the classical LS method, as shown in Table 3 (2 versus 6 lag terms) and in Table 4 (three *versus* six lag terms). For this reason, the first and main advantage of this new method with respect to the classical LS method is its execution speed, which is faster since fewer lag terms are necessary to obtain the flutter speed closest to the *pk* flutter speed.
- The form of the aerodynamic forces approximated with the new method is similar to the form of these forces approximated with the classical LS method. For this reason, a second advantage of this new method is that it can be integrated into the aeroelastic equations of motion in a manner similar to the integration of the LS classical method into these equations.

In this paper we have shown that for an F/A-18 aircraft, the new method developed here, CLS, based on the classical LS method, is very efficient for aeroservoelastic interaction studies.

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## REFERENCES

1. TIFFANY, S.H. and ADAMS, WM. Nonlinear programming extensions to rational function approximation of unsteady aerodynamics, July 1988, NASA TP-2776.
2. EDWARDS, J.W. Unsteady aerodynamic modeling and active aeroelastic control, February 1977, SUDAAR 504, Stanford University, Stanford, CA.
3. ROGER, K.L. Airplane math modeling methods for active control design, August 1977, Structural aspects of active controls AGARD CP-228, pp 4.1-4.11.
4. VEPA, R. Finite state modeling of aeroelastic system, February 1977, NASA CR-2779.
5. KARPEL, M. Design for flutter suppression and gust alleviation using state space modeling, *J Aircr*, 1982, **19**, (3), pp 221-227.
6. TIFFANY, S.H. and ADAMS, W.M. Fitting aerodynamic forces in the Laplace domain: an application of a nonlinear nongradient technique to multilevel constrained optimization, 1984, NASA TM 86317.
7. DUNN, H.J. An analytical technique for approximating unsteady aerodynamics in the time domain, 1980, NASA TP-1738.
8. POIRION, F. Multi-mach rational approximation to generalized aerodynamic forces, *J Aircr*, 1996, **33**, (6), pp 1199-1201.
9. COTOI, I. and BOTEZ, R.M. Method of unsteady aerodynamic forces approximation for aeroservoelastic interactions, *AIAA J Guidance, Control and Dynamics*, 2002, **25**, (5), pp 985-987.
10. COTOI, I., DINU, A.D. and BOTEZ, R.M., Model order reduction for aeroservoelasticity studies by use of LRSRM and LRSM algorithms, 2004, 24th International Congress of Aeronautical Sciences ICAS 2004, Yokohama, Japan, 30 August-3 September, 2004.
11. HILIUTA, A., BOTEZ, R.M. and BRENNER, M. Approximation of unsteady aerodynamic forces by use of fuzzy techniques, 2005, 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Austin, TX, 18-21 April, 2005.
12. GUPTA, K.K. Development and application of an integrated multidisciplinary analysis capability, *Int J for Numerical Methods in Engineering*, 1997, **40**, (3), pp 533-550.