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Aeroservoelasticity Analysis Method Based on an Error Analytical Form Applied on a Business Aircraft

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Abstract: Aeroservoelasticity is a multidisciplinary study that combines the following disciplines: aerodynamics, aeroelasticity and servo-controls. For aeroelasticity studies, the Doublet Lattice Method (DLM) is used to calculate the aerodynamic unsteady forces for a set of reduced frequencies k and Mach numbers M on a business aircraft in the subsonic flight regime. There are three classical methods in the aeroservoelasticity used to approximate these forces $\mathbf{Q}(k, M)$ by rational functions in the Laplace domain $\mathbf{Q}(\bar{s})$: Least Square (LS), Matrix Padé (MP) and Minimum State (MS). A new method called Corrected Least Square (CLS) is presented. This new method uses an analytical form of the error as a function of Laplace variable similar to the analytical form of the aerodynamic forces calculated by use of the LS method. The new CLS method does not take additional time to the computation of the unsteady aerodynamic forces when compared to the LS method, and the aerodynamic forces calculated with the CLS method are closer to the aerodynamic forces data in the frequency domain than the aerodynamic forces calculated by the standard LS method. The new CLS method applied on a business aircraft gives better results (flutter speeds and frequencies) and faster (as it uses smaller number of lag terms) than the LS method.

Key words: Aeroelasticity, aerodynamics, aeroservoelasticity, least squares.

1. INTRODUCTION

For flutter aeroservoelastic analysis, the most common aeroservoelastic software implements the unsteady aerodynamic forces conversion from the frequency $Q(k, M)$ to the Laplace domain $Q(\bar{s})$ by various methods. Each of these aeroservoelastic codes uses one or more of the three methods for unsteady aerodynamic forces conversion: LS, MP or the MS method.

The LS method is used in the following aeroservoelastic codes: Aeroservoelastic Analysis Method for Analog or Digital Systems (ADAM) at Air Force Wright Aeronautical Laboratories (AFWAL), see (Noll et al., 1986), Interaction of Structures Aerodynamics and Controls (ISAC) at NASA Langley Research Center (Adams and Hoadley, 1993) and Structural

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Analysis Routines (STARS) at NASA Dryden Flight Research Center (Gupta, 1997). The LS method uses Pade polynomial approximations. The state space equations include augmented states that represent the aerodynamic lags; their number is dependent on the number of denominator roots in the rational approximation. The entire aerodynamic matrix is approximated by a ratio of matrix polynomials (Vepa, 1977).

In the MP approximation method (Roger et al., 1975), each term of the aerodynamic matrix may be approximated by a polynomial ratio in s. Other modifications of the MP method are suggested by Karpel (1982). In this method, the number of augmented states is equal to the number of the denominator roots.

The capability to enforce or relax the equality constraints was included in the LS, MP and MS methods (Tiffany and Adams, 1988). These capabilities were abbreviated as ELS, EMP and EMS, and they were introduced in the aeroservoelastic computer program called Interaction of Structures, Aerodynamics and Controls (ISAC). The MS approach (MIST) was included in the ASTROS computer program (Chen et al., 1997). This method offers a saving in the number of added states with little or no loss of accuracy in modeling the aerodynamic forces. However, its applicability to the unsteady aerodynamics in the transonic and hypersonic regimes remains to be established. Other aeroservoelastic studies based on the MS method were realized in France at Airbus and Onera (Zimmermann, 1991).

Each of the conversion methods LS, MP and MS present advantages and disadvantages. For example, one advantage of the LS method with respect to the MS method is that the implementation of the LS conversion method is easier to realize and takes smaller execution time. One advantage of the MS method compared to the LS method is that the MS method implies the addition of fewer states than the number of states required by the LS method.

Another approximation method is included in the Flexible Aircraft Modeling Using State Space (FAMUSS) code developed at McDonnell Douglas Co. and later at Boeing (Burnham et al., 2001). This FAMUSS code, used by the Boeing Company in St Louis, is different from the traditional aeroservoelastic LS, MP and MS methods, as rational function aerodynamic fits are not required, such as Pade polynomials from ISAC, ADAM or STARS software. FAMUSS develops a state space model that matches the transfer function frequency response for a given range of frequencies. Fewer equations are obtained than in the rational function approach. Fitting the transfer function frequency response gives a good indication of the difference between the original system and the equivalent system as sensed by the control system. The pole zero model is generated from the transfer function response using linear and nonlinear techniques, and is used to help determine the poles of the system. If the generalized force data is available, then the system poles are directly obtained from them.

Several order reduction algorithms for model reduction implemented in the Robust Control Toolbox were used to verify the new procedure consisting of combinations from linear system theory and Padé approximants (Cotoi and Botez, 2002). These algorithms were the following: Minimal Realization, Schur reduction, Optimal Hankel, Shur's Balanced Stochastic Truncation and Balanced algorithms. The numerical results obtained with the new procedure were compared to the ones obtained with the MS method (weighted and non-weighted). The new procedure gave better performances than the MS method, in terms of error, when the balanced stochastic algorithm was used. The value of the error was found to be 12–40 times smaller than the value of the error obtained with the MS classical method. The disadvantage of this method was its computing time which is higher than the computing time taken by the MS method for these approximations. For the calculations of unsteady aerodynamic forces approximations for any range of reduced frequencies, a combination of fuzzy clustering with shape-preserving techniques was used (Hiliuta et al., 2005). Another method called Mixed State combined the analytical expressions given by the LS and the MS methods (Biskri et al., 2005). This method used a smaller number of lag terms than the classical LS method.

In this paper, a new CLS method is presented, in which the error between the results given by the new method and the results given by the LS method is written under an analytical form similar to the analytical form of the aerodynamic forces calculated by the LS method.

2. EQUATIONS OF MOTION

The free-vibration matrix equation is written as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{1}$$

from which the frequencies and their corresponding vibration mode shapes are calculated. The transformation of the displacements $\mathbf{q}$ into generalized coordinates η is applied by use of the modal shape matrix Φ:

$$\mathbf{q} = \Phi\eta. \tag{2}$$

The equations of motion for a flexible aircraft structure modeled by Finite Element Methods are described by the following matrix equation:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + q_{dyn}\mathbf{A}_e(k)\mathbf{q} = \mathbf{P}(t) \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the generalized mass, damping and stiffness matrices, q_{dyn} is the dynamic pressure expressed as $q_{dyn} = 0.5\rho V^2$, where ρ is the air density and V is the true airspeed, $k = \omega b/V$ is the reduced frequency where ω is the natural frequency, b is the wing semi-chord length, $\mathbf{A}_e(k)$ is the aerodynamic influence coefficient matrix for a given Mach number M and a set of reduced frequencies k values, $\mathbf{q}$ is the displacement vector and $\mathbf{P}(t)$ is the external forcing function.

Equation (2) is replaced into equation (3), and pre-multiplying both sides of equation (3) by Φ^T and Φ, this equation of motion becomes:

$$\hat{\mathbf{M}}\ddot{\eta} + \hat{\mathbf{C}}\dot{\eta} + \hat{\mathbf{K}}\eta + q_{dyn}\mathbf{Q}(k)\eta = \hat{\mathbf{P}}(t) \tag{4}$$

where $\hat{M} = \Phi^T\ \mathbf{M}\ \Phi$, $\hat{C} = \Phi^T\ \mathbf{C}\ \Phi$, and so forth.

Using vibration modal analysis, 44 elastic symmetric modes and 50 elastic anti-symmetric modes with their corresponding frequencies were obtained for a business aircraft. For flutter aeroservoelastic studies, the unsteady aerodynamic generalized forces $\mathbf{Q}$ are calculated using the DLM in NASTRAN code for one Mach number and a range of eight reduced frequencies. Then, flutter speeds and frequencies are calculated. For aeroservoelastic analysis, the equations of motion in the frequency domain (4) are converted in the Laplace domain, and then equation (5) is obtained:

$$\left[\hat{\mathbf{M}}s^2 + \hat{\mathbf{C}}s + \hat{\mathbf{K}} + q_{dyn}\mathbf{Q}_{LS}(\bar{s})\right]\eta(s) = \hat{\mathbf{P}}(s) \tag{5}$$

where $\bar{s} = sb/V$ is the normalized Laplace variable. The generalized aerodynamic forces $\mathbf{Q}(k)$ calculated by the DLM method in equation (4) are approximated in the Laplace domain by use of the LS method (subscript LS), and they are denoted by $\mathbf{Q}_{LS}(\bar{s})$ as shown in equation (5).

3. DESCRIPTION OF THE CLS METHOD

We recall here the approximation of the unsteady aerodynamic forces in the Laplace domain by the LS method $\mathbf{Q}_{LS}(\bar{s})$ under the following form:

$$\mathbf{Q}_{LS}(\bar{s}) = A_0^{LS} + A_1^{LS}\bar{s} + A_2^{LS}\bar{s}^2 + \frac{A_3^{LS}}{\bar{s}+b_1^{LS}}\bar{s} + \frac{A_4^{LS}}{\bar{s}+b_2^{LS}}\bar{s} + \ldots + \frac{A_{n+2}^{LS}}{\bar{s}+b_n^{LS}}\bar{s} \tag{6}$$

where A_i^{LS} are the $n+3$ estimated matrices and b_n^{LS} are the n estimated lag terms.

In the standard LS method, we need to increase the number of lag terms in order to find the best values of the aerodynamic unsteady force approximations in the Laplace domain $Q_{LS}(\bar{s})$ close to their initial values in the frequency domain $Q(k)$. Unfortunately, the increase of lag terms will increase the number of states in the final matrix system, which might have a negative influence on the final aeroservoelastic system behavior.

For this reason, a new method, called the CLS method, is presented. This method minimizes the error between the aerodynamic forces $\mathbf{Q}_{CLS}(\bar{s})$ calculated and their approximation $\mathbf{Q}_{LS}(\bar{s})$ in the Laplace domain. This error is also expressed under the same analytical form as the aerodynamic forces determined by the LS method.

The difference between the unsteady aerodynamic forces calculated in the frequency domain $\mathbf{Q}(k)$ by the DLM and the unsteady aerodynamic forces calculated in the Laplace domain $\bar{s}$, $\mathbf{Q}_{LS}(\bar{s})$, gives the matrix error e_1 for one Mach number:

$$e_1 = \mathbf{Q}(k) - \mathbf{Q}_{LS}(\bar{s} = jk). \tag{7}$$

However, the error matrix e_1 does not have a precise analytical form. We would like to find an analytical form for this matrix, but is not possible for this case.

Therefore, we introduce the error written in the form of an analytical function of $\bar{s}$, denoted as $e(\bar{s})$. This analytical error function is written as a difference between the aerodynamic forces $\mathbf{Q}_{CLS}(\bar{s})$ calculated by the CLS method at a given Mach number M and the aerodynamic forces calculated by the classical LS method, denoted here as $\mathbf{Q}_{LS}(\bar{s})$:

$$e(\bar{s}) = \mathbf{Q}_{CLS}(\bar{s}) - \mathbf{Q}_{LS}(\bar{s}). \tag{8}$$

The error function $e(\bar{s})$ is approximated under the following LS analytical form, which is similar to the analytical form of $\mathbf{Q}_{LS}(\bar{s})$ given by equation (6):

$$e(\bar{s}) = A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \frac{A_3^{err}}{\bar{s} + b_1^{err}}\bar{s} + \frac{A_4^{err}}{\bar{s} + b_2^{err}}\bar{s} + \ldots \frac{A_{n+2}^{err}}{\bar{s} + b_n^{err}}\bar{s} \tag{9}$$

where the superscript err denotes the LS analytical form of the error.

As can be seen in equations (6) and (9), we assume, for notation simplicity, that the same number of lag terms n might be considered in both the LS and CLS methods, which can be written in the following form:

$$b_1^{err} = b_1^{LS}, \quad b_2^{err} = b_2^{LS}, \quad \ldots, \quad b_n^{err} = b_n^{LS}. \tag{10}$$

For each element $e^{r,c}$ of the error matrix (e), a criterion of minimization $J^{r,c}$ is defined as follows:

$$J^{r,c} = \sum_1^l \left[e^{r,c} - \left(A_0^{err}(r,c) + A_1^{err}(r,c)\bar{s} + A_2^{err}(r,c)\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}(r,c)}{\bar{s} + b_i}\bar{s} \right) \right]^2 \tag{11}$$

where r indicates the row index, c indicates the column index and l is the number of reduced frequencies.

Again, for notation simplicity, we neglect the (r, c) superscript in the subsequent equations that demonstrate the new method. The common notation of the lag terms $b_{1,2,3,\ldots}$ is used instead of $b_{1,2,3,\ldots}^{err}$ or $b_{1,2,3,\ldots}^{LS}$ as in equation (10).

In order to minimize the criterion $J^{r,c}$, we set to zero the partial derivatives of $J^{r,c}$ with respect to each unknown factor $A_{0,1,2,\ldots}^{err}(r, c)$. Thus, the following set of equations is written:

$$\frac{\partial J}{\partial A_{0,1,2,\ldots}^{err}} = 0 \tag{12}$$

$$\left\{ \begin{aligned}
&\frac{\partial J}{\partial A_0^{err}} = (-2) * \sum_1^l \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0 \\
&\frac{\partial J}{\partial A_1^{err}} = (-2) * \sum_1^l \bar{s} \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0 \\
&\frac{\partial J}{\partial A_2^{err}} = (-2) * \sum_1^l \bar{s}^2 \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0 \\
&\frac{\partial J}{\partial A_3^{err}} = (-2) * \sum_1^l \left(\frac{\bar{s}}{\bar{s} + b_1} \right) \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0 \\
&\frac{\partial J}{\partial A_4^{err}} = (-2) * \sum_1^l \left(\frac{\bar{s}}{\bar{s} + b_2} \right) \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0 \\
&\ldots \\
&\frac{\partial J}{\partial A_{n+2}^{err}} = (-2) * \sum_1^l \left(\frac{s}{s + b_n} \right) \left[e - \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s} + b_i}\bar{s} \right) \right] = 0
\end{aligned} \right. \tag{13}$$

where $*$ is the scalar multiplication operator. Equation (13) is equivalent to the following system of equations

$$\begin{cases} \sum_1^l e = \sum_1^l \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \\ \sum_1^l \bar{s} * e = \sum_1^l \bar{s} \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \\ \sum_1^l \bar{s}^2 * e = \sum_1^l \bar{s}^2 \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_1}\right) \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_2}\right) \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \\ \dots \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} e = \sum_1^l \left(\frac{\bar{s}}{\bar{s}+b_n}\right) \left(A_0^{err} + A_1^{err}\bar{s} + A_2^{err}\bar{s}^2 + \sum_{i=1}^n \frac{A_{i+2}^{err}}{\bar{s}+b_i}\bar{s} \right) \end{cases} \tag{14}$$

and further to the following system of equations:

$$\begin{cases} \sum_1^l e = A_0^{err} \sum_1^l 1 + A_1^{err} \sum_1^l \bar{s} + A_2^{err} \sum_1^l \bar{s}^2 + \sum_1^n \left[A_{n+2}^{err} \left(\sum_1^l \frac{\bar{s}}{\bar{s}+b_i} \right) \right] \\ \sum_1^l \bar{s} * e = A_0^{err} \sum_1^l \bar{s} + A_1^{err} \sum_1^l \bar{s}^2 + A_2^{err} \sum_1^l \bar{s}^3 + \sum_1^n \left[A_{n+2}^{err} \left(\sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} \right) \right] \\ \sum_1^l \bar{s}^2 * e = A_0^{err} \sum_1^l \bar{s}^2 + A_1^{err} \sum_1^l \bar{s}^3 + A_2^{err} \sum_1^l \bar{s}^4 + \sum_1^n \left[A_{n+2}^{err} \left(\sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} \right) \right] \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} e = A_0^{err} \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} + A_1^{err} \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} + A_2^{err} \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} \\ \qquad + \sum_1^n \left[A_{n+2}^{err} (\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_1)}) \right] \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} e = A_0^{err} \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} + A_1^{err} \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} + A_2^{err} \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} \\ \qquad + \sum_1^n \left[A_{n+2}^{err} \left(\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_2)} \right) \right] \\ \dots \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} e = A_0^{err} \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} + A_1^{err} \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} + A_2^{err} \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} \\ \qquad + \sum_1^n \left[A_{n+2}^{err} \left(\sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_n)} \right) \right]. \end{cases} \tag{15}$$

The system of equations (15) is equivalent to the following system of equations:

$$b = Ax \tag{16}$$

where:

$$b = \left[\sum_1^l e \quad \sum_1^l \bar{s} e \quad \sum_1^l \bar{s}^2 e \quad \sum_1^l \frac{\bar{s}}{\bar{s} + b_1} e \quad \sum_1^l \frac{\bar{s}}{\bar{s} + b_2} e \quad \ldots \quad \sum_1^l \frac{\bar{s}}{\bar{s} + b_n} e \right]^T \tag{17}$$

and

$$A = \begin{bmatrix} \sum_1^l 1 & \sum_1^l \bar{s} & \sum_1^l \bar{s}^2 & \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} \\ \sum_1^l \bar{s} & \sum_1^l \bar{s}^2 & \sum_1^l \bar{s}^3 & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} \\ \sum_1^l \bar{s}^2 & \sum_1^l \bar{s}^3 & \sum_1^l \bar{s}^4 & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} & \cdots & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_1} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_1)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_1)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_1)} \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_2} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_2)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_2)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_1^l \frac{\bar{s}}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^2}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^3}{\bar{s}+b_n} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_1)(\bar{s}+b_n)} & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_2)(\bar{s}+b_n)} & \cdots & \sum_1^l \frac{\bar{s}^2}{(\bar{s}+b_n)(\bar{s}+b_n)} \end{bmatrix} \tag{18}$$

so that the solution of the system x is obtained from equation (16):

$$x = A^{-1} b = \left[A_0^{err} \quad A_1^{err} \quad A_2^{err} \quad A_3^{err} \quad A_4^{err} \quad \ldots \quad A_{n+2}^{err} \right]^T. \tag{19}$$

Equation (8) gives the unsteady aerodynamic forces calculated by the new method, denoted by $\mathbf{Q}_{CLS}(\bar{s})$, as follows:

$$\mathrm{Q}_{CLS}(\bar{s}) = \mathrm{Q}_{LS}(\bar{s}) + e(\bar{s}). \tag{20}$$

We replace $\mathbf{Q}_{LS}(\bar{s})$ and $\mathbf{e}(\bar{s})$ given by equations (6) and (9) into equation (20) and then $\mathbf{Q}_{CLS}(\bar{s})$ is:

$$\begin{aligned} Q_{CLS}(\bar{s}) \quad &= \quad \left(A_0^{LS} + A_0^{err}\right) + \left(A_1^{LS} + A_1^{err}\right)\bar{s} + \left(A_2^{LS} + A_2^{err}\right)\bar{s}^2 \\ &+ \quad \sum_{i=1}^{n} \frac{\bar{s}}{\bar{s} + b_i^{LS}} \left(A_{i+2}^{LS} + A_{i+2}^{err}\right). \end{aligned} \tag{21}$$

Thus, equation (21) can be written as follows:

Table 1. Reduction rates of the aerodynamic forces (LS *vs* CLS) calculated for one to six lag terms (44 symmetric and 50 anti-symmetric modes).

No of lag terms	Real aerodynamic Force Q_R reduction rate		Imaginary aerodynamic force Q_I reduction rate		Total aerodynamic force Q reduction rate	
	44 modes Symmetric	50 modes Anti-symmetric	44 modes Symmetric	50 modes Anti-symmetric	44 modes Symmetric	50 modes Anti-symmetric
1	47.8557%	53.2161%	55.0565%	55.6118%	52.9452%	55.6701%
2	40.0813%	50.1881%	40.8778%	49.2132%	40.7686%	49.7256%
3	50.1427%	47.9628%	32.9445%	43.1553%	44.6819%	46.1208%
4	85.0945%	69.3996%	78.7497%	68.0849%	82.4504%	69.0531%
5	99.9798%	34.9087%	99.9745%	47.8878%	99.9775%	42.9524%
6	96.3031%	87.7367%	96.2231%	88.6993%	96.4076%	88.7839%

$$Q_{CLS}(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + \sum_{i=1}^{n} \frac{A_{i+2}}{\bar{s} + b_i^{LS}}\bar{s} \tag{22}$$

where: $A_0 = A_0^{LS} + A_0^{err}$, $A_1 = A_1^{LS} + A_1^{err}$, $A_2 = A_2^{LS} + A_2^{err}, \ldots, A_{n+2} = A_{n+2}^{LS} + A_{n+2}^{err}$.

4. RESULTS PRESENTATION

The two approximation methods, LS and the new method denoted by CLS, were applied to a business aircraft for the following flight condition: Mach number $M = 0.88$ and 8 reduced frequencies $k = 0.001$, 0.1, 0.3, 0.5, 0.7, 0.9, 1.1 and 1.4.

In order to compare the results obtained by the new method with the results from the *LS* method, the reduction rate defined by: $100 * abs\ (Q_{LS} - Q_{CLS})\,/Q_{LS}$ was used, where *abs* is the absolute value function.

Table 1 shows the numerical values of the reduction rate of the aerodynamic forces (real, imaginary and total) for 1 to 6 lag terms and for symmetric and anti-symmetric aircraft modes. The results shown in Table 1 are graphically presented in Figures 1 and 2.

In Figures 3 and 4, the execution time is presented for the standard LS method versus the CLS method. Figure 3 shows the execution time for the aerodynamic force calculations by the LS versus the CLS method for the 44 symmetric aircraft modes for 1 to 6 lag terms, while Figure 4 shows the execution time for the aerodynamic force calculations by the LS versus the CLS method for the 50 anti-symmetric aircraft modes for 1 to 6 lag terms. These figures show that the use of the new method does not require additional time for the computation of unsteady aerodynamic forces.

In general, the corrective part of the CLS method is expressed as the difference between the maximum execution time obtained with the LS method and the maximum execution time obtained with the CLS method. Figure 5 shows that this corrective part takes a maximum of 1 second for a maximum of 6 lag terms. Most of the execution time is taken by the LS method.

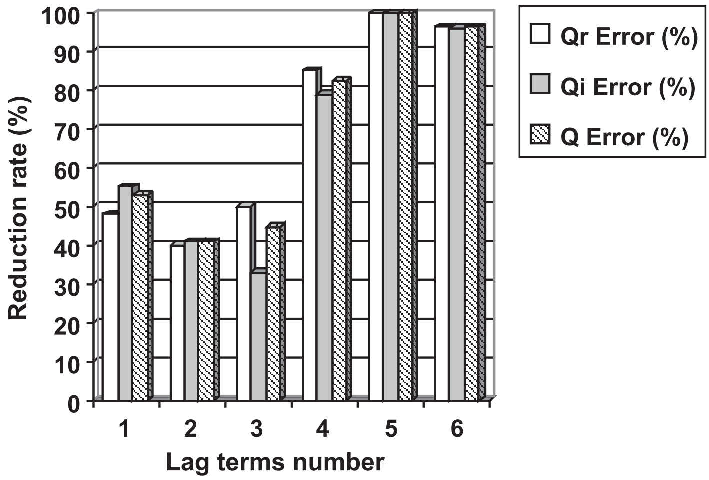

Figure 1. Reduction rates of the real, imaginary and total aerodynamic forces (LS *vs* CLS) of the 44 symmetric modes for 1 to 6 lag terms.

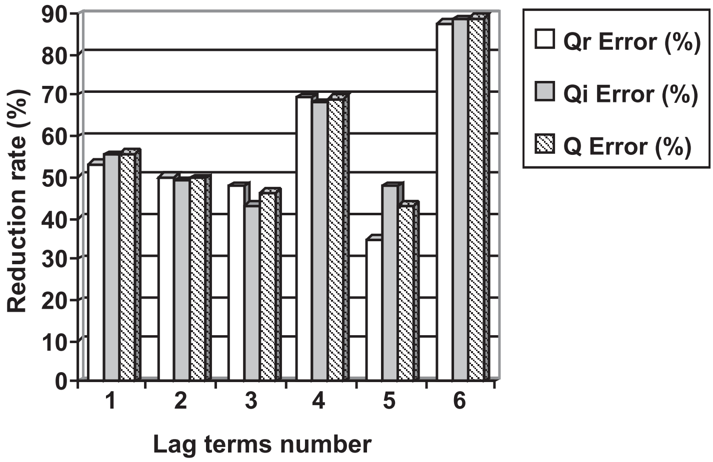

Figure 2. Reduction rates of the real, imaginary and total aerodynamic forces (LS *vs* CLS) of the 50 anti-symmetric modes versus the total number of lag terms.

Figure 6 shows the comparison between two aerodynamic force elements chosen randomly such as Q(35,15) and Q(38,25), calculated by use of the DLM method (denoted here by ‘Data’), and their approximations by use of the LS method and the new CLS method, for two different number of lag terms (1 and 4).

Tables 2 and 3 show the results presented in terms of flutter speeds and frequencies with respect to standard flutter results for aircraft symmetric modes (Table 2) and anti-symmetric modes (Table 3). A number of lag terms from 1 to 4 are considered in the new method.

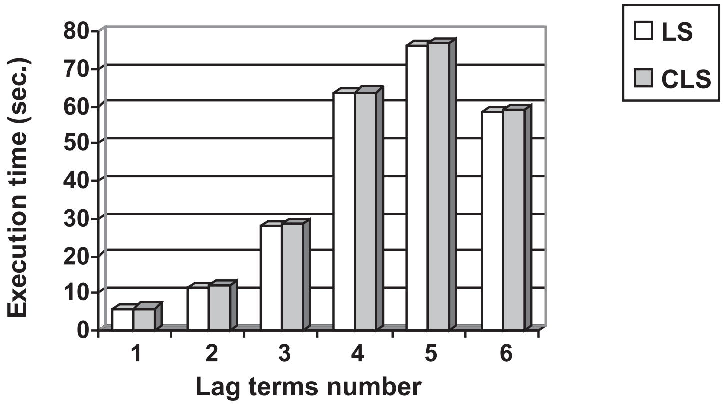

Figure 3. Execution time (seconds) of the LS and the new CLS method for the 44 symmetric modes for 1 to 6 lag terms.

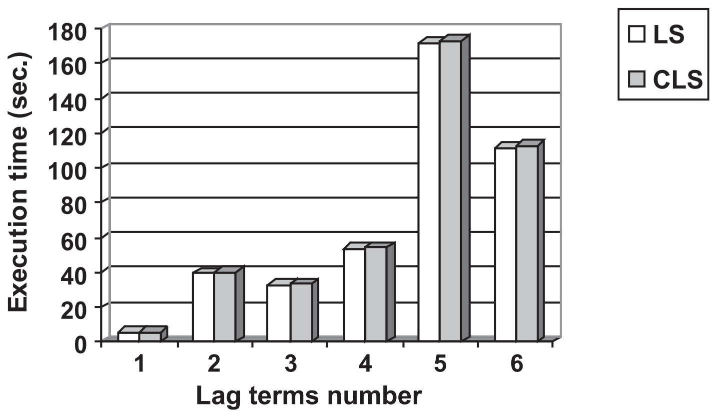

Figure 4. Execution time (seconds) of the LS and the new CLS method for the 50 anti-symmetric modes for 1 to 6 lag terms.

5. DISCUSSION OF RESULTS

As seen in Table 1 and its associated Figures 1 and 2, the aerodynamic force reduction rate is much smaller with the new CLS method compared to the standard LS method. The reduction rate for the error ranges from 33% to 99%. Figures 3, 4 and 5 show that the execution time of the new method is almost the same as that of the LS method. Figure 6 shows the aerodynamic data forces obtained by the LS and the new CLS approximations. In all of the monitored cases, we noticed that with the CLS method the aerodynamic forces are closer to the aerodynamic force data in the frequency domain $\mathbf{Q}(k)$ and that they have better shapes than the shapes generated by the standard LS method.

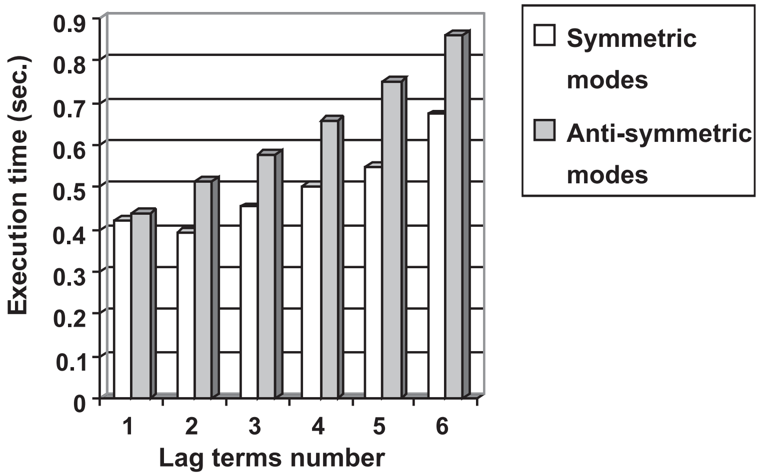

Figure 5. Execution time (seconds) of the error estimation for the symmetric and anti-symmetric modes.

Table 2. Flutter results obtained with the LS and CLS new methods with respect to the standard flutter results for the 44 symmetric modes of the aircraft.

	Flutter #1	Flutter #2	Flutter #3	Flutter #4
pk-LS (8 lags)	Speed: 0.015%	Speed: 0.644%	Speed: 0.018%	Speed: 0.298%
	Freq: 2.232%	Freq: 0.339%	Freq: 0.223%	Freq: 0.053%
pk-CLS (1 lag)	Speed: 4.142%	Speed: 4.211%	Speed: 4.089%	Speed: 0.707%
	Freq: 1.599%	Freq: 15.606%	Freq: 29.534%	Freq: 0.181%
pk-CLS (2 lags)	Speed: 0.230%	Speed: 0.893%	Speed: 1.934%	Speed: 0.044%
	Freq: 1.268%	Freq: 15.933%	Freq: 35.367%	Freq: 0.0005%
pk-CLS (3 lags)	Speed: 3.304%	Speed: 6.118%	Speed: 1.895%	Speed: 0.0001%
	Freq: 1.394%	Freq: 1.642%	Freq: 0.261%	Freq: 0.032%
pk-CLS (4 lags)	Speed: 0.204%	Speed: 0.351%	Speed: 0.301%	Speed: 0.014%
	Freq: 0.233%	Freq: 0.078%	Freq: 0.117%	Freq: 0.041%

In general, to obtain good results, the traditional LS method requires a large number of lag terms, such as the 8 lag terms used here. For example, we observe from Tables 2 and 3 that we obtain the best results by use of the new CLS method with 4 lag terms and by use of the LS method with 8 lag terms.

The first advantage of the new CLS method is that the final form of the aerodynamic force approximations with the CLS method is similar to the LS form, which means that the introduction of the new CLS method into the aeroelastic general equation is similar to the introduction of the LS method into the same type of equation. Therefore, for users of the standard LS method, the CLS method is easy to integrate in computer programs. Indeed, it is enough to add the corrective part represented by $e(\bar{s})$ to a LS computer program in order for it to become a CLS program. Thus, to pass from the LS form to the CLS form, only few minor modifications are required.

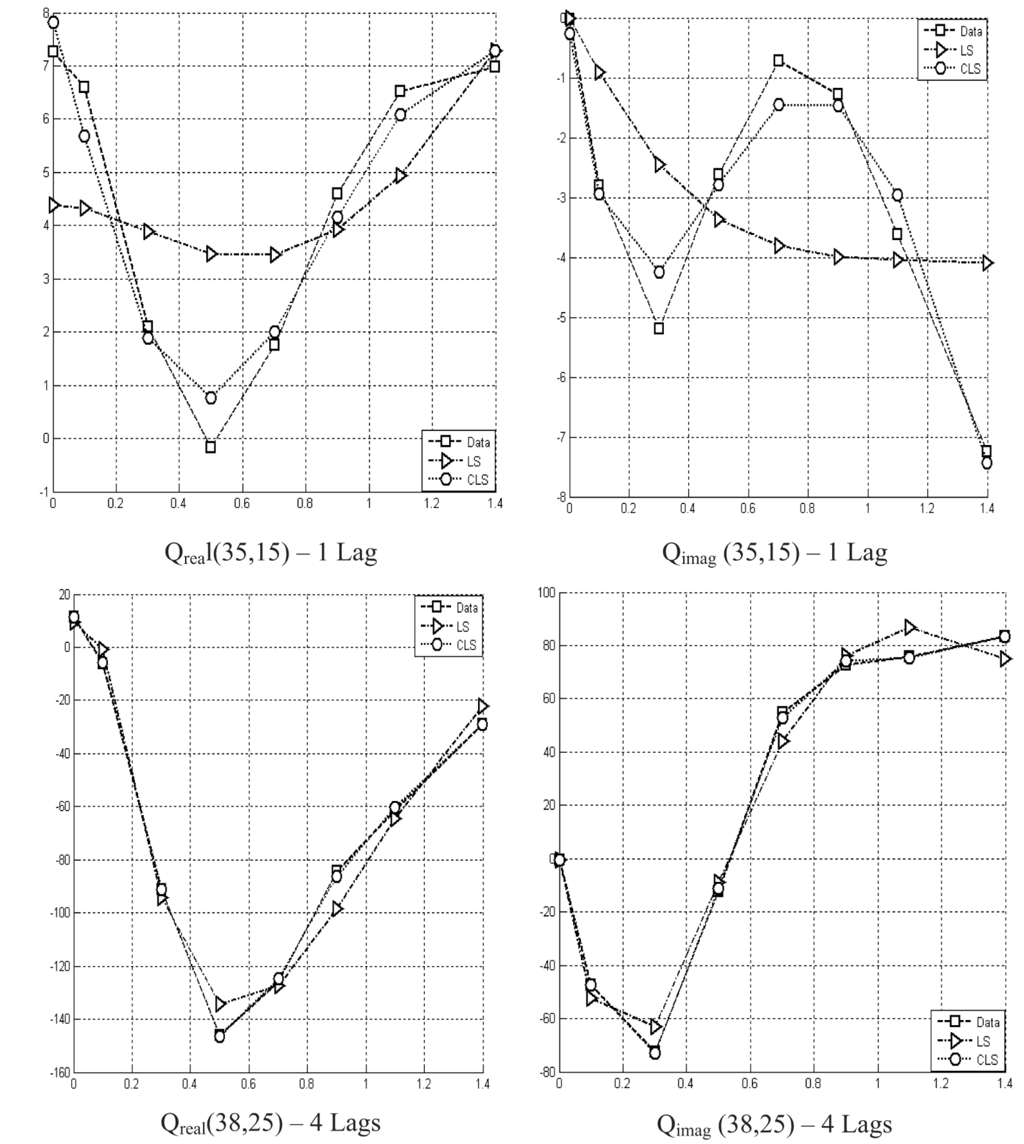

Figure 6. Aerodynamic force elements $\mathbf{Q}$(35,15) and $\mathbf{Q}$(38,25) versus reduced frequency k range for 1 and 4 lag terms, respectively.

The second advantage of this method is not to be dismissed, namely that the approximation error is evaluated and added to the final error form. This advantage can be seen on the plots of the aerodynamic forces where a better quality of approximation for a small number of lag terms can be observed. By analyzing these graphics as well as the tables of flutter values, we can see the reduction of the error of the aerodynamic forces $\mathbf{Q}(\bar{s})$ with the CLS method compared to the LS method.

Table 3. Flutter results obtained with the LS and CLS new methods with respect to the standard flutter results for the 50 antisymmetric modes of the aircraft.

	Flutter #1	Flutter #2	Flutter #3	Flutter #4
pk-LS (8 lags)	Speed: 0.15%	Speed: 0.563%	Speed: 1.243%	Speed: 1.737%
	Freq: 0.161%	Freq: 0.112%	Freq: 0.092%	Freq: 0.940%
pk-CLS (1 lag)	Speed: 1.633%	Speed: 0.822%	Speed: 5.477%	Speed: 1.563%
	Freq: 0.537%	Freq: 0.922%	Freq: 0.176%	Freq: 0.147%
pk-CLS (2 lags)	Speed: 2.013%	Speed: 0.706%	Speed: 5.016%	Speed: 2.450%
	Freq: 0.273%	Freq: 0.877%	Freq: 0.204%	Freq: 0.503%
pk-CLS (3 lags)	Speed: 0.491%	Speed: 0.096%	Speed: 0.326%	Speed: 2.501%
	Freq: 0.088%	Freq: 0.032%	Freq: 0.022%	Freq: 0.569%
pk-CLS (4 lags)	Speed: 0.092%	Speed: 0.680%	Speed: 0.328%	Speed: 1.897%
	Freq: 0.082%	Freq: 0.192%	Freq: 0.0357%	Freq: 0.493%

The third advantage can be seen in the execution speed. In fact, with fewer lag terms and more precision, the CLS method, compared to the LS method, gives us better results in less time. Finally, this new CLS method seems to be extremely efficient for aeroservoelastic interactions studies.

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